Radian Measure

These notes are intended as a summary of section 6.3 (p. 487 – 493) in your workbook. You should also read the section for more complete explanations and additional examples.

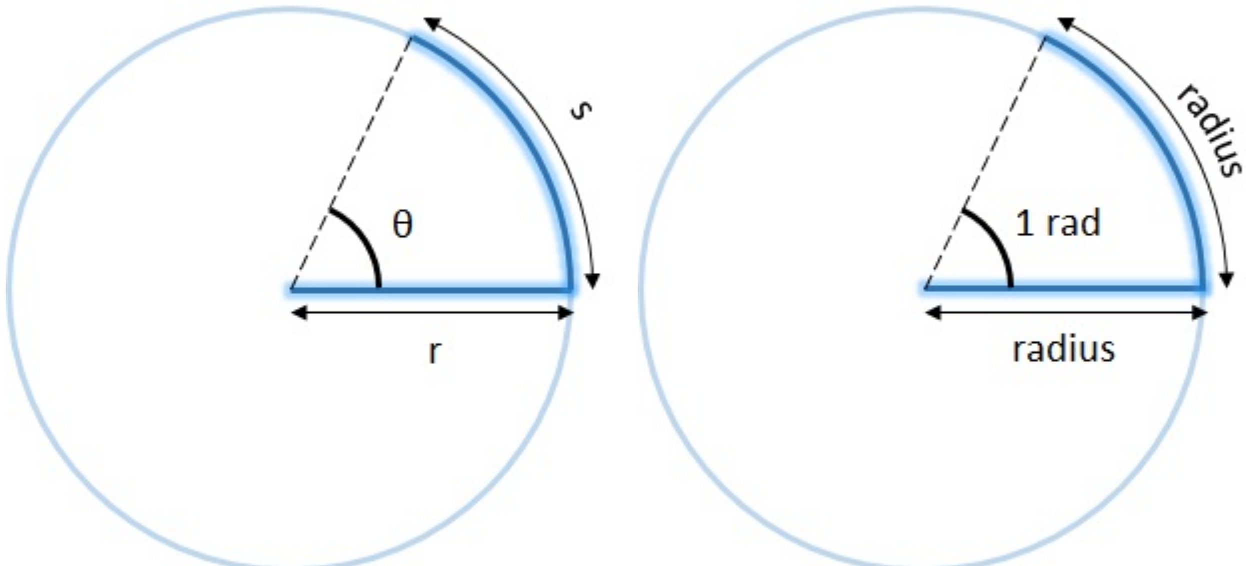
Introduction

Measurements require units. For example, when we measure the length of an object, we might measure it in meters. Most measurements can be expressed in a variety of units. Length, for example, can be expressed in meters, yards, inches, miles, etc.

So far, when measuring angles, we have expressed them in degrees. However, like other measurements, angles can be expressed in a variety of units. The radian is an example of another unit for measuring angles.

Definition

One **radian** is defined as the angle subtended by a circular arc when the arc length is equal to the radius.



More generally, the angle in radians is equal to

- the arc length divided by the radius.

$$\theta = \frac{s}{r}$$

- the arc length of an arc on the unit circle (radius = 1).

For a full circle, the arc length is the circumference ($2\pi r$). If we apply the definition of the radian, we can determine the angle subtended:

$$\begin{aligned}\theta &= \frac{s}{r} \\ &= \frac{2\pi r}{r} \\ \theta &= 2\pi\end{aligned}$$

Thus, there are 2π radians in a full circle. In other words, an angle of 2π radians is equal to an angle of $360°$.

Converting between Radians and Degrees

We can use the fact that 2π radians is equal to $360°$ to convert any angle in degrees into an angle in radians (or vice versa). If we divide both sides of $2\pi = 360°$ by 2, we can see that

$$\pi = 180°$$

To convert degrees to radians:

$$radians = degrees \times \frac{\pi}{180°}$$

Note: Angles in radians are often expressed as exact values (fractions involving π).

To convert radians to degrees:

$$degrees = radians \times \frac{180°}{\pi}$$

Example 1 (sidebar p. 489)

a) Given $\theta = 170°$, determine its measure in radians. Give the exact measure and its approximate value to the nearest hundredth.

b) Given $\theta = -\frac{12\pi}{11}$ radians

i) Sketch the angle in standard position.

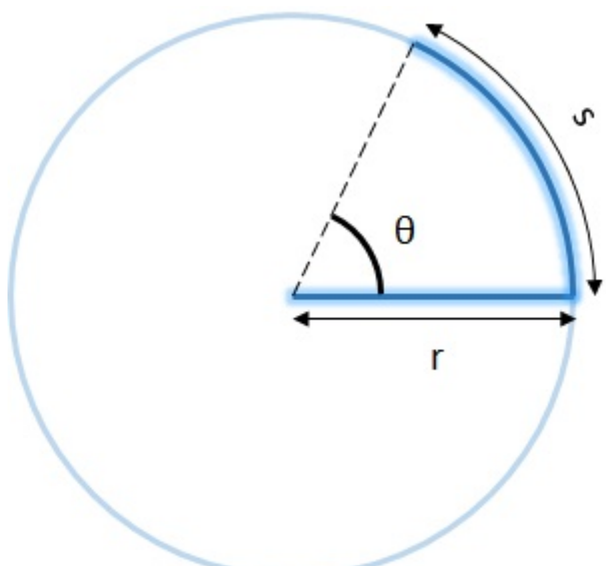
ii) Determine its measure to the nearest tenth of a degree.

c) Given $\theta = 7.5$ radians, determine its measure to the nearest tenth of a degree.

Arc Length

When the angle θ is expressed in radians, the arc length can be determined using:

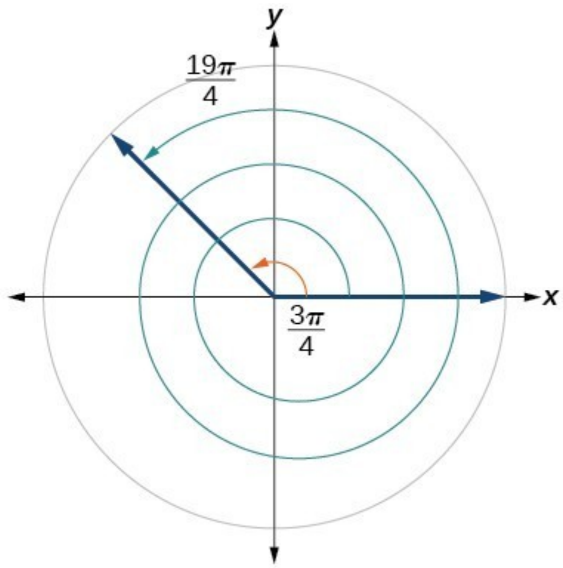
$$s = \theta \cdot r$$



Coterminal Angles

Coterminal angles do not depend on the units used to measure them. In radians, the measures of angles that are coterminal with θ are found by adding or subtracting multiples of 2π radians (because $2\pi = 360°$).

For example, $\frac{3\pi}{4}$ and $\frac{19\pi}{4}$ are coterminal.



$$\frac{19\pi}{4} = \frac{3\pi}{4} + \frac{16\pi}{4}$$
$$= \frac{3\pi}{4} + 4\pi$$

$$\frac{19\pi}{4} = \frac{3\pi}{4} + 2(2\pi)$$

The measure of any angle coterminal with θ can be written in radians as:

$$\theta + 2\pi k$$

where k is an integer ($k \in \mathbb{Z}$).

Example (not in workbook)

a) Determine the measures of all the angles in standard position between -4π and 4π that are coterminal with an angle of $\frac{\pi}{3}$ in standard position. Sketch the angles.

b) Write an expression for all the angles that are coterminal with an angle of $\frac{\pi}{3}$ in standard position.

Homework: #4, 5, 7, 8, 11 in the exercises (p. 494 – 501). Answers on p. 501.